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Solitary waves in nonlinear dispersive systems with zero average dispersion

S. K. Turitsyn,[1] N. F. Smyth,[2] and E. G. Turitsyna[3]

[1]*Institut für Theoretische Physik I, Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, Germany*

[2]*Department of Mathematics and Statistics, University of Edinburgh, The King's Buildings, Mayfield Road, Edinburgh EH9 3JZ, Scotland, United Kingdom*

[3]*Institute for Nonlinear Studies, 630090 Novosibirsk, Russia*

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The dynamics of a solitary wave in a nonlinear lossy system with varying dispersion and periodic amplification is examined. It is demonstrated that in contrast to the traditional soliton model in which the average dispersion balances the nonlinearity, in a nonlinear system with varying dispersion, stable pulse propagation is possible even if the average dispersion is zero. As a practical example, we demonstrate dispersion-managed soliton transmission in a cascaded optical amplifier system at zero average dispersion. The possibility of transmitting a soliton with finite energy at zero dispersion (when timing jitter is suppressed) is very attractive for practical applications. [S1063-651X(98)50307-8]

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A nonlinear solitary wave is an example of a stable wave packet that preserves its shape during propagation in a nonlinear dispersive medium (see, e.g., [1–5], and references therein). As is well known, this stability is provided by a balance between the effects of dispersion and nonlinearity. Based on this rather general idea of compensating dispersive broadening by nonlinearity, the soliton concept has been used successfully in a wide range of physical and practical applications. Interest in soliton theory has additionally been stimulated by the discovery and development of the inverse scattering transform, which allows powerful mathematical tools to be applied to the investigation of solitons in a variety of physical applications. Integrable soliton systems model important physical phenomena, for instance, optical signal propagation in fiber transmission lines. Much progress has already been made in the further development of soliton theory in this context. Recent developments in lightwave communication systems have created opportunities for new applications of and developments in soliton theory [6–19]. In particular, new and interesting soliton dynamics takes place if the nonlinearity and dispersion vary periodically with distance down the fiber [8,9]. A solitary wave can survive such a structural perturbation, although the pulse has features that make it drastically different from a traditional soliton. Basic soliton properties such as invariant (during propagation) shape and power, determined by a balance between average nonlinearity and average dispersion, have to be reconsidered in the case of large variations of the dispersion. Recently discovered dispersion-managed solitons [6–19] occur as a result of a complex balance among the effects of the varying local dispersion, fiber loss, periodic amplification, nonlinearity, and residual dispersion. Interplay between these factors leads to a rich variety of possible transmission regimes. The pulse's dynamics shows rapid oscillations of its power and width in the compensation period and slow evolution on longer scales due to fiber nonlinearity and residual dispersion [9]. Numerical solutions have revealed the following features of the dispersion-managed soliton. The shape of the evolving pulse (in its central region) is not always the sech profile of the nonlinear Schrödinger equation (NLSE) soliton, but varies from a sech profile to a Gaussian shape and further to a sinclike profile with the change of the map strength. The evolving pulse is chirped (i.e., the pulse phase has a nontrivial time dependence). The energy of the stable breathing pulse is well above that of the NLSE soliton with the same pulse width when the fiber has the corresponding average dispersion. Due to the chirp of the pulse the interaction between two neighboring dispersion-managed solitons is suppressed as a result of the fast rotation of the relative phase. Numerical modeling of the lossless system has shown that stable solitonlike propagation is possible along a fiber with zero and even normal average dispersion [13]. This regime clearly demonstrates a principal difference between a dispersion-managed (DM) soliton and the regular fundamental soliton. A DM soliton is then a new kind of information carrier whose features differ significantly from that of the fundamental soliton. Indeed, the properties of the DM soliton mean that it forms a base for a new paradigm for soliton-based transmission in optical fiber links.

The fundamental soliton has energy proportional to the fiber dispersion and inversely proportional to the pulse width (see, e.g., [2,5,20]). To keep the signal-to-noise ratio large enough (as required for good system performance), one needs to operate the fiber not too close to the zero-dispersion point. On the other hand, a jitter in pulse arrival times, which results from the Gordon-Haus effect [21], is proportional to the fiber dispersion. To overcome timing jitter, it is advantageous to transmit solitons at wavelengths close to the zero-dispersion point. It is then clear that it would be desirable to produce a finite-energy soliton pulse that can propagate in a fiber with low dispersion. Operation at very low average dispersion allows for the reduction of timing jitter for a number of channels. This is of crucial importance for wavelength-division multiplexing transmission that is part of the key method of increasing transmission capacity of optical fiber lines.

In this Rapid Communication we examine DM optical soliton transmission in amplified fiber lines at zero average dispersion. Note that the obtained results are rather general and can be applied to many soliton-bearing systems. While some investigation has already been done in the lossless situation [13], soliton dynamics in a fiber with zero average dispersion has not yet been considered for the important case in which the amplification distance is of the same order as the compensation period. The approach developed in our previous work [9,11,19] (see also [10,12,17,18]) allows us to describe the dynamics of the DM soliton and to obtain the dependence of its average energy and chirp on the pulse width. We have found that for a breathing soliton propagating in a system with zero net dispersion there exists a minimal possible average energy for a fixed strength of the dispersion map for the soliton to be stable. Using scaling properties of the model, we extend our results to a range of similar dispersion maps.

The dynamics of a breathing DM soliton down a fiber line is governed by the nonlinear Schrödinger equation with periodic coefficients

$$iA_z + d(z)A_{tt} + c(z)|A|^2A = 0. \tag{1}$$

Here $d(z) = \lambda_0^2 D(z)L/(4\pi C_e t_0^2)$ [C_e is the speed of light, t_0 is a parameter (10 ps in the present paper), D is the dispersion coefficient measured in ps/(nm×km), and $c(z) = P_0 L \sigma^{(1,2)} \exp(-2L\gamma z)$]. Here we assume two-step dispersion management and the upper indices (1,2) correspond to the first and second fibers, respectively. The quantity $\sigma^{(1,2)}$ is the corresponding nonlinear coefficient and γ describes fiber losses, assumed equal for both fibers. The propagation distance z is normalized by the dispersion compensation period L and time t is normalized by the parameter t_0; the envelope of the electric field $E = E(T,Z)$ is scaled by the power parameter P_0. The amplification distance can be different from the compensation period in general. The chromatic dispersion $d(z) = \tilde{d}(z) + \langle d \rangle$ represents the sum of a rapidly varying (over one compensation period) high local dispersion and a constant residual dispersion ($\langle d \rangle \ll \tilde{d}$). In this Rapid Communication we consider (without loss of generality) the specific example of a transmission line that consists of fibers with anomalous dispersion [$D^{(1)} = D = 3.57$ ps/(nm×km)] and with normal dispersion [$D^{(2)} = -D = -3.57$ ps/(nm ×km)] of equal lengths of 40 km. The amplification distance is equal to the compensation period $Z_a = L = 80$ km. The evolution equation (1) was then solved numerically using a pseudospectral method [22] and the following parameter values were taken. The fiber attenuation was chosen as 0.21 dB/km in both fiber sections and the in-line amplifiers were assumed to compensate for the loss between two consecutive amplifiers, which gives an amplifier gain of 16.8 dB. The nonlinear refractive index was set to $n_2 = 3.2 \times 10^{-20}$ m^2/W. Finally, the effective fiber area was taken to be $A_{eff} = 55$ μm^2 for both fibers.

As was previously shown in [9–12,17,19] the evolution of the central (energy-bearing) part of the DM pulse can be approximately described by a system of ordinary differential equations (ODEs) for the pulse width and chirp. This approach is very useful for the approximate determination of the optimal input signal. The validity of this optimal input signal can then be verified from numerical solutions of the full equation.

The fast (over one period) dynamics of the central part of the dispersion-managed soliton is given, to leading order, by $A(z,t) = |A|\exp(i\phi)$ with

$$|A(z,t)|^2 = \frac{|Q(x)|^2}{T(z)},$$

$$\frac{\partial \phi}{\partial t} = 2\,\frac{M(z)t}{T(z)} \tag{2}$$

(see [9,11,19], and references therein for details). Here $x = t/T(z)$ and the evolution of $T(z)$ and $M(z)$ is given by

$$\frac{dT}{dz} = 4d(z)M,$$

$$\frac{dM}{dz} = \frac{d(z)C_1}{T^3} - \frac{c(z)C_2}{T^2}; \tag{3}$$

C_1 and C_2 are constants related to the pulse shape through $C_1 = \int |Q_x|^2 dx / (\int x^2 |Q|^2 dx)$ and $C_2 = \int |Q|^4 dx / (4 \int x^2 |Q|^2 dx)$. The lumped action of the amplifiers is accounted for by the transformation of the pulse power at the junctions corresponding to the locations of the amplifiers. Stationary soliton propagation corresponds to the periodic solutions of Eqs. (3). We have found that periodic solutions exist when the path-average dispersion is zero and even normal. Because soliton transmission at the zero-dispersion point is very interesting for practical applications due to the suppressed timing jitter, in what follows without loss of generality, we focus on this case. Qualitatively, the possibility of balancing nonlinear and dispersive effects at zero or normal dispersion results from the large variation of the soliton width during one compensation period and the self-similar structure of the DM soliton given by Eq. (2). The total phase shift can be found from $[d(TM)]/dz = 4dM^2 + [C_1 d(z)]/T^2 - [C_2 c(z)]/T$. Here, TM is proportional to the rapidly oscillating part of the soliton phase in accordance with Eq. (2). The requirement for the recovery of the soliton (DM pulse) phase (except linear growth) at the end of the compensation section leads to the condition $\langle d(z)[4M^2 + C_1/T^2] \rangle = \langle [C_2 c(z)]/T(z) \rangle$. The physical interpretation of this condition is obvious: The quantity $4M^2 + C_1/T^2 = \Omega^2$ is nothing more than the square of the spectral bandwidth Ω of a chirped pulse. The total phase shift due to pulse chirping, dispersion, and nonlinearity should be zero (balanced) on average for true periodic propagation. It can easily be seen that the requirement of the anomalous average dispersion $\langle d \rangle > 0$, which provides for the existence of the traditional NLSE soliton, is replaced for the DM soliton by the condition $d_{eff} = \langle d\Omega^2 \rangle > 0$, which can be satisfied at zero and even normal path-averaged dispersion.

First we demonstrate that periodic breathing-like soliton propagation is indeed possible even if the path-average dispersion is zero. The fast evolution of the pulse width and peak power in the system, with zero average dispersion, is shown in Figs. 1 and 2, respectively. Evidently, in contrast to the lossless case studied in [13], in the present system power evolution is highly asymmetrical due to fiber losses. The peak power $P(z)$ at any point within the compensation cell is related to the power after amplifier P_a by the relationship

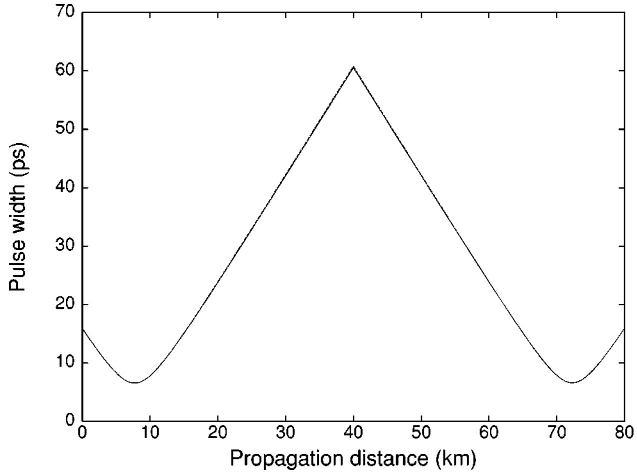

FIG. 1. Evolution of the pulse width of a dispersion-managed soliton in a system with zero average dispersion over one compensation period. The two minima correspond to the chirp-free points that are optimal for the launching of a transform-limited pulse into the system. The solid line corresponds to the full numerical solution of (1) and the dashed line (which can hardly be distinguished from the solid line) corresponds to the solution of the approximate ODEs.

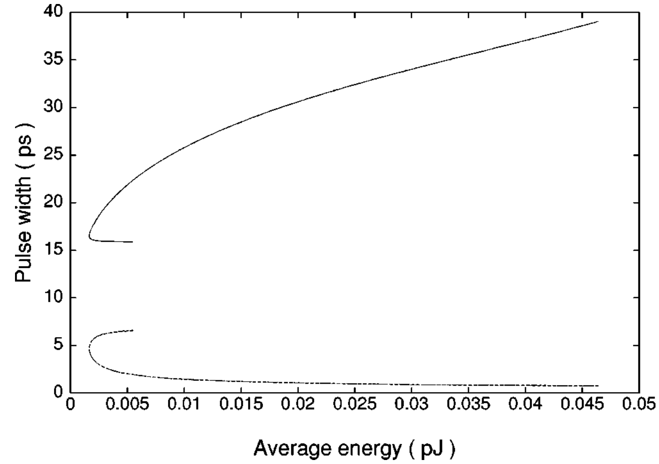

FIG. 3. Pulse widths taken at the two chirp-free points plotted as functions of the average soliton energy. The solid line is for the width of the chirped pulse at the beginning of the section and the dashed line is for the pulse widths at the first and second chirp-free points, respectively. It can be seen that there exists a minimal average energy of the dispersion-managed soliton propagating at zero average dispersion.

$P(Z)=P_a\exp(-2\gamma Z)/T(Z)$. Figures 1 and 2 also show a comparison of the solutions of the approximate ODEs and Eq. (1). Solid lines correspond to the full numerical solution of Eq. (1) and the dashed lines (which can hardly be distinguished from the solid lines) correspond to the solution of the approximate ODEs. It should be noted that in contrast to a conventional soliton, a dispersion-managed pulse experiences breathing-like oscillations of its width over the amplification period. Therefore, to describe the dependence of the pulse width on the system parameters we introduce as an appropriate characteristic pulse width the width at the chirp-free points of the map. At these points (local minima of the pulse width in Fig. 1) a Fourier-limited pulse can be launched into the system. At other points (including the amplifier locations), the launched soliton needs additional prechirping. An analytical method for finding the locations of the chirp-free points is presented in [16]. However, this approach cannot be applied to a system with zero average dispersion. In this Rapid Communication we use a formalism developed in [9,18] to determine the locations of the chirp-free points along the map. There are only two optimal launch points for a given map. In Fig. 3 the dependence of the average soliton energy on the pulse width is plotted. The pulse widths are taken at the two chirp-free points that correspond to the minima of the varying pulse width in Fig. 1. The solid line is for the pulse width at the beginning of the section (where the soliton is chirped) and the dashed line is for the first and second chirp-free points. It can be seen that the widths are about the same at these optimal launch points and cannot be distinguished within the resolution of Fig. 3. An important observation from Fig. 3 is that, in contrast to

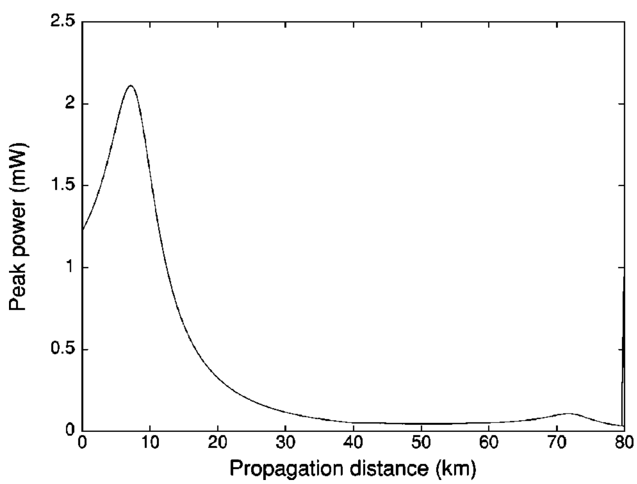

FIG. 2. Peak power evolution along the compensation cell. The two maxima of the peak power correspond to the minimal pulse widths in the section. Peak power is recovered at the end of the section by an optimal amplifier. The comparison of the ODE and PDE is shown to be similar to Fig. 1.

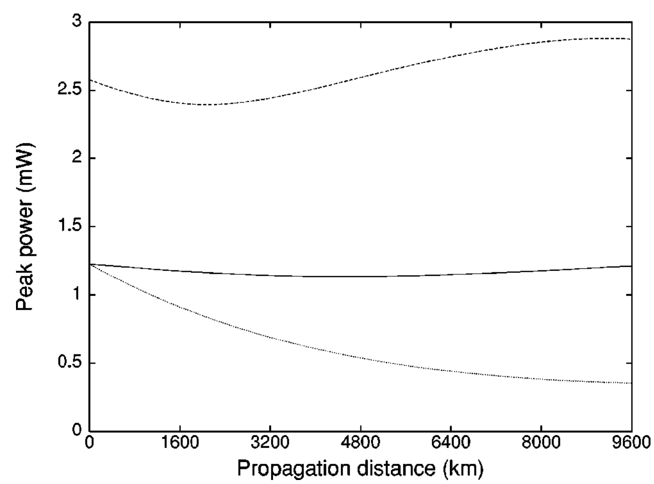

FIG. 4. Comparison of the evolution of peak power of the input Gaussian pulse with optimal initial parameters, corresponding to the dispersion-managed soliton (solid line), an unchirped input pulse (dotted line), and an input pulse with arbitrary power, but with the optimal chirp (dashed line).

the conventional fundamental soliton, in a system with zero average path dispersion, there exists a minimum energy for fixed system parameters. This dependence is rather different from the empirical formula found in [8] for the energy of a dispersion-managed soliton in a system with anomalous average dispersion.

Next let us consider the long-term pulse evolution. We have verified that choosing an input Gaussian pulse with optimal parameters determined from the ODEs leads to substantial improvement in soliton transmission. One can alternatively launch a chirp-free soliton at the optimal points or, by appropriate phase modulation, launch a pulse with the required chirp at other points. In Fig. 4 the evolution of the peak power of the different input pulses is shown. The solid line is for an initial pulse with power and chirp equal to that of the dispersion-managed soliton of the same width, the dotted line is for an input pulse with the same power and width, but without chirp, and the dashed line is for an input pulse with the same width and chirp as the dispersion-managed soliton, but with different power. It can be seen that stable propagation can be achieved with a maximal fitting of the input pulse parameters with those of the dispersion-managed soliton corresponding to the given map. In particular, an appropriate chirp given by an external modulator can reduce the chromatic dispersion penalty relative to the chirpless case. Alternatively, a chirp-free pulse can be launched at one of the optimal points of the map [8,16].

It is worth noting that the results obtained above can be applied to the problem of the generation of stretched pulses in additive mode-locking laser systems [6]. Indeed, a stretched pulse in an additive mode-locking laser system was suggested before the idea of using dispersion-managed solitons in optical fiber transmission. The similarity that exists between these two problems allows the results obtained for soliton transmission to be applied to the generation of stretched pulses in mode-locking laser systems. In particular, we also note that in the case of zero net dispersion, there exist scalings that can be used to obtain optimal pulses for other dispersion maps. For instance, changing the local dispersions as $D_{new}=\kappa D=\kappa D_{old}$, we obtain periodic solutions similar to those found in this Rapid Communication, but with a pulse width $T_{new}=\sqrt{\kappa}T_{old}$, energy $E_{new}=\sqrt{\kappa}E_{old}$, and chirp parameter $M_{new}=\kappa^{-1}M_{old}$.

In conclusion, we have examined the propagation of a chirped breathing soliton in a dispersion-managed optical amplifier system with zero average dispersion. We have found that for a fixed strength of the map there exists a minimal energy for such a soliton. Local (over one period) evolution of the soliton peak power in the system considered is asymmetrical, in contrast to lossless systems. The results obtained are further confirmation of the great potential of WDM soliton transmission in the realization of future ultra-large capacity communication systems.

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